

# **Aircraft**

## **Basic Science**

**Eighth Edition**

**Michael J. Kroes**

**James R. Rardon (Deceased)**

**Michael S. Nolan**



New York Chicago San Francisco Lisbon London Madrid Mexico City  
Milan New Delhi San Juan Seoul Singapore Sydney Toronto

Copyright © 2013 by The McGraw-Hill Companies, Inc. All rights reserved. Except as permitted under the United States Copyright Act of 1976, no part of this publication may be reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior written permission of the publisher.

ISBN: 978-0-07-179918-8

MHID: 0-07-179918-4

The material in this eBook also appears in the print version of this title: ISBN: 978-0-07-179917-1, MHID: 0-07-179917-6.

McGraw-Hill eBooks are available at special quantity discounts to use as premiums and sales promotions, or for use in corporate training programs. To contact a representative please e-mail us at [bulksales@mcgraw-hill.com](mailto:bulksales@mcgraw-hill.com).

All trademarks are trademarks of their respective owners. Rather than put a trademark symbol after every occurrence of a trademarked name, we use names in an editorial fashion only, and to the benefit of the trademark owner, with no intention of infringement of the trademark. Where such designations appear in this book, they have been printed with initial caps.

Information has been obtained by McGraw-Hill from sources believed to be reliable. However, because of the possibility of human or mechanical error by our sources, McGraw-Hill, or others, McGraw-Hill does not guarantee the accuracy, adequacy, or completeness of any information and is not responsible for any errors or omissions or the results obtained from the use of such information.

#### TERMS OF USE

This is a copyrighted work and The McGraw-Hill Companies, Inc. ("McGraw-Hill") and its licensors reserve all rights in and to the work. Use of this work is subject to these terms. Except as permitted under the Copyright Act of 1976 and the right to store and retrieve one copy of the work, you may not decompile, disassemble, reverse engineer, reproduce, modify, create derivative works based upon, transmit, distribute, disseminate, sell, publish or sublicense the work or any part of it without McGraw-Hill's prior consent. You may use the work for your own noncommercial and personal use; any other use of the work is strictly prohibited. Your right to use the work may be terminated if you fail to comply with these terms.

THE WORK IS PROVIDED "AS IS." MCGRAW-HILL AND ITS LICENSORS MAKE NO GUARANTEES OR WARRANTIES AS TO THE ACCURACY, ADEQUACY OR COMPLETENESS OF OR RESULTS TO BE OBTAINED FROM USING THE WORK, INCLUDING ANY INFORMATION THAT CAN BE ACCESSED THROUGH THE WORK VIA HYPERLINK OR OTHERWISE, AND EXPRESSLY DISCLAIM ANY WARRANTY, EXPRESS OR IMPLIED, INCLUDING BUT NOT LIMITED TO IMPLIED WARRANTIES OF MERCHANTABILITY OR FITNESS FOR A PARTICULAR PURPOSE. McGraw-Hill and its licensors do not warrant or guarantee that the functions contained in the work will meet your requirements or that its operation will be uninterrupted or error free. Neither McGraw-Hill nor its licensors shall be liable to you or anyone else for any inaccuracy, error or omission, regardless of cause, in the work or for any damages resulting therefrom. McGraw-Hill has no responsibility for the content of any information accessed through the work. Under no circumstances shall McGraw-Hill and/or its licensors be liable for any indirect, incidental, special, punitive, consequential or similar damages that result from the use of or inability to use the work, even if any of them has been advised of the possibility of such damages. This limitation of liability shall apply to any claim or cause whatsoever whether such claim or cause arises in contract, tort or otherwise.

# Contents

Preface to the Eighth Edition *ix*  
Preface to the Seventh Edition *xi*

## **1. Fundamentals of Mathematics** 1

Arithmetic 1  
Algebra 10  
Geometry 15  
Trigonometry 23  
Alternative Number Systems 25  
Charts and Graphs 26  
Review Questions 32

## **2. Science Fundamentals** 33

Measurements 33  
Gravity, Weight, and Mass 35  
Force and Motion 37  
Centrifugal and Centripetal Force 39  
Composition and Resolution of Forces 40  
Work, Energy, and Power 41  
Machines 42  
Heat 45  
Heat Transfer 47  
Fluids 49  
The Nature and Laws of Gases 51  
Sound 52  
Review Questions 55

## **3. Basic Aerodynamics** 57

Physical Properties of the Air 57  
Airfoils 63  
Drag 66  
High-Speed Flight 69  
Review Questions 77

## **4. Airfoils and Their Applications** 79

Airfoil Profiles 79  
Performance of Airfoils 81  
Shapes and Dimensions of Airfoils 86  
Stalls and Their Effects 97  
Review Questions 103

<b>5. Aircraft in Flight</b>	<b>105</b>
Forces on the Airplane in Flight	105
Aircraft Stability	108
Axes of the Airplane	108
Aircraft Control	112
Aircraft Design Variations	117
Airfoils on Biplanes	119
Helicopters	122
Helicopter Controls	128
Helicopter Configurations	132
Review Questions	136
<b>6. Aircraft Drawings</b>	<b>137</b>
Types of Drawings	137
Drafting Techniques	140
Review Questions	156
<b>7. Weight and Balance</b>	<b>157</b>
Fundamental Principles	157
Weight-and-Balance Terminology	160
Determination of the EWCG Location	166
Aircraft Modifications	175
Loading the Airplane	177
Extreme Weight-and-Balance Conditions	178
Loading Conditions	180
Simplified Loading Methods	180
Calculating Weight and Balance for Large Aircraft	182
Weight and Balance for a Helicopter	183
Light Sport Aircraft	187
Weight-Shift-Control Aircraft	187
Review Questions	188
<b>8. Aircraft Materials</b>	<b>189</b>
Aircraft Materials	189
Properties of Materials	190
General Properties of Metals	195
Alloys	196
Corrosion	199
Fatigue	199
Aircraft Metals	200
Plastics	206
Composite Materials	207
Aircraft Wood	210
Aircraft Fabrics	211
Review Questions	212
<b>9. Metal Fabrication Techniques and Processes</b>	<b>213</b>
Mill Products	213
Fabrication of Metal Components	215
Heat Treatment of Metals	217

Hardness Testing 222  
Nondestructive Inspection 223  
Corrosion Control 227  
Finish and Surface-Roughness Symbols 231  
Metal Surface Treatments 233  
Review Questions 233

## **10. Standard Aircraft Hardware 235**

Standards 235  
Specifications 237  
Threaded Fasteners 237  
Nonthreaded Fasteners 246  
Panel and Cowling Fasteners 253  
Cable Fittings 253  
Turnbuckles 254  
Safety Belts 256  
Keep Current 257  
Review Questions 257

## **11. Hand Tools and Their Application 259**

Measurement and Layout 259  
Wrenches 268  
Screwdrivers 274  
Pliers 276  
Hammers 278  
Cutting Tools 278  
Punches 288  
Safety Equipment 288  
Review Questions 289

## **12. Aircraft Fluid Lines and Fittings 291**

Types of Fluid-Line Systems 291  
Fabrication, Repair, and Installation of Fluid Lines 301  
Review Questions 312

## **13. Federal Aviation Regulations 313**

History of the Federal Aviation Administration 313  
Aviation Safety Regulation 314  
Organization of the FAA 314  
Federal Aviation Regulations 315  
Certification of Products and Parts 322  
Airworthiness Standards 327  
Operations Certification 332  
Review Questions 337

## **14. Technical Publications 339**

Advisory Circulars 339  
Service-Difficulty Reporting Program 339  
Airworthiness Directives 340  
Type Certificate Data Sheets 346

Supplemental Type Certificates 357  
Manufacturers' Publications 358  
Technical Manuals 358  
Manufacturers' Operating Publications 363  
Review Questions 378

**15. Ground Handling and Safety 379**

General Safety Precautions 379  
Compressed-Gas Safety 379  
Fire Safety 380  
Flight-Line Safety 381  
Towing Aircraft 382  
Taxiing and Starting 383  
Tying Down or Mooring Aircraft 388  
Jacking and Hoisting Aircraft 397  
Ground-Support Equipment 402  
Fueling 405  
Review Questions 410

**16. Aircraft Inspection and Servicing 411**

Required Aircraft Inspections 411  
Large and Turbine-Powered Multiengine Airplanes 416  
Conducting a 100-Hour or Annual Inspection 419  
Lubrication and Servicing 423  
Servicing Aircraft 425  
Operational Inspection 427  
Maintenance Records 427  
Malfunction or Defect Report 429  
Cleaning Aircraft and Parts 429  
Review Questions 432

**Appendix 433**

**Glossary 453**

**Index 457**

# Fundamentals of Mathematics **1**

## INTRODUCTION

The science of mathematics, so important to the modern age of technology, had its beginnings in the dim ages of the past. It is probable that prehistoric people recognized the differences in quantities at an early age and therefore devised methods for keeping track of numbers and quantities. In the earliest efforts at trade it was necessary for the traders to figure quantities. For example, someone might have traded ten sheep for two cows. To do this, the trader had to understand the numbers involved.

As time progressed, the ancient Babylonians and Egyptians developed the use of mathematics to the extent that they could perform marvelous engineering feats. Later the Greeks developed some of the fundamental laws which are still in use today. One of the great Greek mathematicians was a philosopher named Euclid, who prepared a work called *Elements of Geometry*. This text was used by students of mathematics for almost 2000 years. Another Greek mathematician was Archimedes, who is considered one of the greatest mathematicians of all time. One of his most important discoveries was the value of  $\pi$  (pi), which is obtained by dividing the circumference of a circle by its diameter. Archimedes discovered many other important mathematical relationships and also developed the early study of calculus. Modern differential and integral calculus were discovered by Sir Isaac Newton in the seventeenth century. These discoveries are considered some of the most important in the history of mathematics.

Today's modern technology, including aircraft maintenance, is greatly dependent upon mathematics. Computing the weight and balance of an aircraft, designing a structural repair, or determining the serviceability of an engine part are but a few examples of an aviation maintenance technician's need for mathematics. Electronic calculators and computers have made mathematical calculations more rapid and usually more accurate. However, these devices are only as good as the information put into them and do not excuse the technician from learning the fundamentals of mathematics.

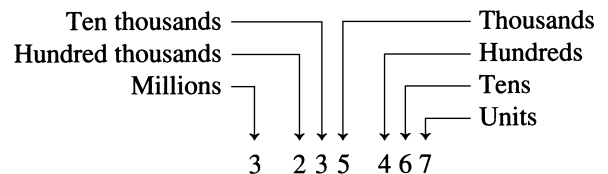
It is expected that you, the aviation technician/student, have taken or are taking mathematics courses that go beyond the material in this chapter. The purpose of this chapter is to refresh your understanding of fundamental mathematical processes. Emphasis is placed on those mathematical terms or problems that you will encounter in portions of your technical studies or employment.

## ARITHMETIC

### Numbers

The 10 single-number characters, or **numerals**—1, 2, 3, 4, 5, 6, 7, 8, 9, and 0—are called **digits**. Any number may be expressed by using various combinations of these digits. The arrangement of the digits and the number of digits used determine the value of the number being expressed.

Our number system is called a **decimal** system, the name being derived from the Latin word *decem*, meaning "ten." In the decimal system the digits are arranged in columns, which are powers of 10. The column in which a certain digit is placed determines its expressed value. When we examine the number 3 235 467, we indicate the column positions as follows:



We may analyze the total number by considering the values expressed by each column, thus:

Units	7	7
Tens	6	60
Hundreds	4	400
Thousands	5	5 000
Ten thousands	3	30 000
Hundred thousands	2	200 000
Millions	3	3 000 000

We may now observe that the total number consists of 3 millions, 2 hundred thousands, 3 ten thousands, 5 thousands, 4 hundreds, 6 tens, and 7 units. The total number is read “three million, two hundred thirty-five thousand, four hundred sixty-seven.”

There are several classes of numbers. **Whole numbers**, also called **integers**, are those which contain no fractions. Examples of such numbers are 3, 10, 250, and 435. A **fraction** is a part of a unit. A **mixed number** contains a whole number and a fraction. An **even number** is one which is divisible by 2. The numbers 2, 4, 6, 8, 10, 48, and 62 are even. **Odd numbers** are those which are not divisible by 2. The numbers 3, 5, 11, 13, 53, and 61 are odd.

## Addition and Subtraction

Addition and subtraction may be considered the simplest of mathematical operations; however, these operations require practice to do quickly and accurately.

### Addition

Addition is the process of combining the values of two or more numbers into a single value. The combined value is called the **sum** of the values (numbers). The sign for addition is the **plus sign** (+). This sign placed between numbers indicates that they are to be added. Numbers to be added may be arranged horizontally or vertically in columns, as shown here:

$$324 + 25 + 78 = 427$$

$$\begin{array}{r} 324 \\ 25 \\ + 78 \\ \hline 427 \end{array}$$

Numbers to be added are usually arranged in columns for more speed and convenience in performing the addition.

$$\begin{array}{r} 7 \\ 6 \\ 3 \\ 8 \\ + 5 \\ \hline 29 \end{array} \quad \begin{array}{r} 32 \\ 420 \\ 8 \\ 19 \\ 26 \\ + 248 \\ \hline 753 \end{array} \quad \begin{array}{r} 4382 \\ 276 \\ 1820 \\ 2753 \\ 47 \\ + 238 \\ \hline 9516 \end{array}$$

Practice is one of the surest ways to learn to add accurately and rapidly. If you want to attain proficiency, you should take time to make up problems or find problems already prepared and then practice solving the problems until you feel comfortable.

It is recommended that you practice adding by sight. It is quite easy to learn to add by sight when the numbers to be added contain only one digit. With a little practice, the sight of any two digits will immediately bring the sum to mind. Thus when seeing the digits 6 and 5, for example, you should immediately think *11*, or upon seeing 9 and 7, you should instantly think *16*.

When we want to add two-digit numbers by sight, it is merely necessary to add the units and then the tens.

Suppose that the numbers 45 and 23 are presented for addition. The units are 5 and 3, so we immediately think *8 units*. The tens are 4 and 2, so we think *6 tens*. The sum of 6 tens and 8 units is 68. If the units in an addition add to a sum greater than 9, we must remember to add the ten or tens to the sum of the tens. If we wish to add 36 and 57, for example, we see that the units add to 13, or 1 ten and 3 units. We record the 3 units and **carry** the ten, adding it to the 3 tens and 5 tens. The result is 9 tens and 3 units, or 93.

### Subtraction

Subtraction is the reverse of addition. The sign for subtraction is the minus sign (-). In ordinary arithmetic a smaller number is always subtracted from a larger number.

In subtraction the number from which another is to be subtracted is called the **minuend**, the number being subtracted from the other is called the **subtrahend**, and the result is called the **difference**.

$$\begin{array}{r} 675 \quad \text{minuend} \\ -342 \quad \text{subtrahend} \\ \hline 333 \quad \text{difference} \end{array}$$

In subtraction it is important to remember the components of a number, that is, the units, tens, hundreds, and so on. This will make it easier to perform the necessary operations. In the preceding example, the numbers in the subtrahend are smaller than the corresponding numbers in the minuend, and the operation is simple. If a number in the minuend is smaller than the corresponding number in the subtrahend, it is necessary to **borrow** from the next column. For example,

$$\begin{array}{r} 853 \\ -675 \\ \hline 178 \end{array}$$

In the first column we find the 3 smaller than the 5, and therefore we must borrow 1 ten from the next column. We then subtract 5 from 13 to obtain 8. We must remember that there are only 4 tens left in the second column, and we have to borrow 1 hundred from the next column to make 140. We subtract 70 from 140 and obtain 70, and so we place a 7 in the tens column of the answer. Since we have borrowed 1 hundred from the 8 hundreds of the third column, only 7 hundreds are left. We subtract 6 hundreds from 7 hundreds, thus leaving 1 hundred. We therefore place a 1 in the hundreds column of the answer.

### Multiplication

The act of **multiplication** may be considered multiple addition. If we add 2 + 2 to obtain 4, we have multiplied 2 by 2, because we have taken 2 two times. Likewise, if we add 2 + 2 + 2 + 2 to obtain 8, we have multiplied 2 by 4, because we have taken 2 four times.

In multiplication the number to be multiplied is called the **multiplicand**, and the number of times the multiplicand is to be taken is called the **multiplier**. The answer obtained



from a multiplication is the **product**. The following example illustrates these terms:

$$\begin{array}{r}
 425 \text{ multiplicand} \\
 \cdot 62 \text{ multiplier} \\
 \hline
 850 \\
 2550 \\
 \hline
 26350 \text{ product}
 \end{array}$$

Note that the terms *multiplicand* and *multiplier* may be interchanged. For example,  $2 \times 4$  is the same as  $4 \times 2$ .

When we use multiplication to solve a specific problem, the names of the terms have more significance. For example, if we wish to find the total weight of 12 bags of apples and each bag weighs 25 pounds (lb), then the multiplicand is 25 and the multiplier is 12. We then say 12 times 25 lb is 300 lb, or  $12 \times 25 = 300$ .

We can understand the multiplication process by analyzing a typical but simple problem, such as multiplying 328 by 6.

$$\begin{array}{r}
 328 \\
 \times 6 \\
 \hline
 48 \quad = 6 \times 8 \\
 120 \quad = 6 \times 20 \\
 1800 \quad = 6 \times 300 \\
 \hline
 1968
 \end{array}$$

In actual practice we do not write down each separate operation of the multiplication as shown in the foregoing problem, but we shorten the process by carrying figures to the next column. In the problem shown we can see that  $6 \times 8$  is 48 and that the 4 goes into the tens column. Therefore, when we multiply, we merely carry the 4 over and add it to the next multiplication, which is in the tens column. When we use this method, the operation is as follows:

$$\begin{array}{r}
 14 \\
 328 \\
 \times 6 \\
 \hline
 1968
 \end{array}$$

The first step in this operation is to multiply 8 by 6.

$$6 \times 8 = 48$$

Record the 8 (units) and carry the 4 (tens), then multiply 2 by 6.

$$6 \times 2 = 12$$

Add the 4 to obtain 16. Record the 6 (tens) and carry the 1 (hundreds). Then multiply 3 by 6.

$$6 \times 3 = 18$$

Add the 1 and obtain 19. Record the 19.

When there is more than one digit in the multiplier, we repeat the process for each digit, but we must shift one column to the left for each digit. This is because the right-hand digit of the multiplier is units, the next digit to the left is tens,

the next is hundreds, and so on. If we multiply 328 by 246, we proceed as follows:

$$\begin{array}{r}
 328 \\
 \times 246 \\
 \hline
 1968 \\
 1320 \\
 6560 \\
 \hline
 80688
 \end{array}$$

Zeros were placed at the end of the second and third multiplications in the first example to show that we were multiplying by 40 and 200, respectively. In actual practice the zeros are not usually recorded. In the preceding multiplication we multiplied 328 first by 6, then by 40, and finally by 200. When we added these products, we obtained the answer, 80 688.

Accurate multiplication requires great care. First, it is important to know the multiplication tables. Second, care must be taken to record products in the correct column. Third, the addition must be made carefully and accurately. In order to acquire proficiency in multiplication, practice is essential.

In any mathematical problem it is smart to check the answer for accuracy. There are a number of methods for checking multiplication, and the most obvious is to divide the product by either the multiplicand or the multiplier. If the product is divided by the multiplicand, the quotient (answer) should be the multiplier.

Another method for checking multiplication is to repeat the problem, reversing the multiplicand and multiplier. If the product is the same in each case, the answer is probably correct.

## Division

The act of **division** may be considered the reverse of multiplication; that is, division is the separating or dividing of a number into a certain number of equal parts. The symbol for division is the division sign ( $\div$ ), and it is read "divided by." For example,  $98 \div 4$  is read "98 divided by 4." In arithmetic there are two commonly used methods for the division of whole numbers. These are **short division** and **long division**. The terms used to describe the elements of a division problem are **dividend**, which is the number to be divided; **divisor**, the number of times the dividend is to be divided; and **quotient**, the number of times the divisor goes into the dividend. In the problem  $235 \div 5 = 47$ , the number 235 is the dividend, 5 is the divisor, and 47 is the quotient.

The process of short division is often used to divide a number by a divisor having only one digit. This is accomplished as follows:

$$\begin{array}{r}
 3 \\
 7 \overline{)3857} \\
 \underline{51}
 \end{array}$$

The first step is to divide 38 by 7. Since  $7 \times 5 = 35$ , it is obvious that after the division of 38 by 7 there will be a remainder of 3. This 3 is held over in the hundreds column and becomes the first digit of the next number to be divided. This number is 35, and 7 goes into 35 five times without leaving a remainder. The only number left to divide is the 7, into which the divisor goes once. The quotient is thus 551.

The process of division as just explained may be understood more thoroughly if we analyze the numbers involved. The dividend 3857 may be expressed as  $3500 + 350 + 7$ . These numbers divided separately by 7 produce the quotients 500, 50, and 1. Adding these together gives 551, which is the quotient obtained from the short division.

Long division is employed most often when the dividend and the divisor both contain more than one digit. The process is somewhat more complex than that of short division, but with a little practice, long division may be accomplished easily and accurately.

To solve the problem  $18\,116 \div 28$ , we arrange the terms of the problem as shown here:

$$\begin{array}{r} 647 \\ 28 \overline{)18116} \\ \underline{168} \phantom{0} \\ 131 \phantom{0} \\ \underline{112} \phantom{0} \\ 196 \phantom{0} \\ \underline{196} \\ 0 \end{array}$$

The first step in solving the problem is to divide 181 by 28, because 181 is the smallest part of the dividend into which 28 can go. It is found that 28 will go into 181 six times, with a remainder of 13. The number 168 ( $6 \times 28$ ) is placed under the digits 181 and is subtracted. The number 13, which is the difference between 168 and 181, is placed directly below the 6 and 8 as shown, and then the number 1 is brought down from the dividend to make the number 131. The divisor 28 will go into 131 four times, with a remainder of 19. The final digit 6 of the dividend is brought down to make the number 196. The divisor 28 will go into 196 exactly seven times. The quotient of the entire division is thus 647.

If we study the division shown in the foregoing example, we will find that the dividend is composed of  $28 \times 600 = 16\,800$ ,  $28 \times 40 = 1120$ , and  $28 \times 7 = 196$ . Then by adding  $16\,800 + 1120 + 196$ , we find the sum, which is 18 116, the original dividend. We could divide each part of the dividend by 28 separately to obtain 600, 40, and 7 and then add these quotients together; however, it is usually quicker and simpler to perform the divisions as shown.

If a divisor does not go into a dividend an even number of times, there will be a remainder. This remainder may be expressed as a whole number, a fraction, or a decimal. Fractions and decimals are discussed later in this chapter.

In the following example the divisor will not go into the dividend an even number of times, so it is necessary to indicate a remainder:

$$\begin{array}{r} 223\frac{10}{16} \\ 16 \overline{)3578} \\ \underline{32} \phantom{0} \\ 37 \phantom{0} \\ \underline{32} \phantom{0} \\ 58 \phantom{0} \\ \underline{48} \phantom{0} \\ 10 \end{array}$$

## Fractions

A **fraction** may be defined as a part of a quantity, unit, or object. For example, if a number is divided into four equal parts, each part is one-fourth ( $\frac{1}{4}$ ) of the whole number. The parts of a fraction are the **numerator** and the **denominator**, separated by a line indicating division.

In Fig. 1-1 a rectangular block is cut into four equal parts; each single part is  $\frac{1}{4}$  of the total. Two of the parts make  $\frac{1}{2}$  the total, and three of the parts make the fraction  $\frac{3}{4}$  of the total.

A fraction may be considered an indication of a division. For example, the fraction  $\frac{3}{4}$  indicates that the numerator 3 is to be divided by the denominator 4. One may wonder how a smaller number, such as 3, can be divided by a larger number, such as 4. It is actually a relatively simple matter to accomplish such a division when we apply it to a practical problem. Suppose we wish to divide 3 gallons (gal) of water into four equal parts. Since there are 4 quarts (qt) in a gallon, we know that there are 12 qt in 3 gal. We can then divide the 12 qt into four equal parts of 3 qt each. Three quarts is  $\frac{3}{4}$  gal; thus we see that 3 divided by 4 is equal to  $\frac{3}{4}$ . The principal fact to remember concerning fractions is that a fraction indicates a division. The fraction  $\frac{1}{2}$  means that 1 is to be divided by 2, or that the whole is to be cut in half.

A fraction whose numerator is less than its denominator is called a **proper fraction**. Its value is less than 1. If the numerator is greater than the denominator, the fraction is called an **improper fraction**.

A **mixed number** is a combination of a whole number and a fraction, such as  $32\frac{2}{3}$  and  $325\frac{23}{33}$ , which mean  $32 + \frac{2}{3}$  and  $325 + \frac{23}{33}$ .

Fractions may be changed in form without changing their values. If the numerator and the denominator of a fraction are both multiplied by the same number, the value of the fraction remains unchanged, as shown in the following example:

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

The value of  $\frac{9}{12}$  is the same as  $\frac{3}{4}$ . In a similar manner, the value of a fraction is not changed if both the numerator and the denominator are divided by the same number.

$$\frac{24 \div 12}{36 \div 12} = \frac{2}{3}$$

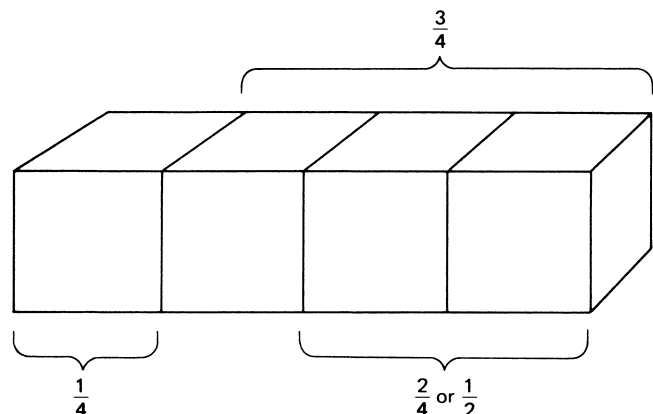


FIGURE 1-1 Fractions of a whole.

Thus we see that a large fraction may be simplified in some cases. This process is called **reducing the fraction**. To reduce a fraction to its lowest terms, we divide both the numerator and the denominator by the largest number that will go into each without leaving a remainder. This is accomplished as follows:

$$\frac{36}{40} \div 4 = \frac{9}{10} \quad \text{and} \quad \frac{525}{650} \div 25 = \frac{21}{26}$$

### Addition and Subtraction of Fractions

In order to add or subtract fractions, the denominators of the fractions must have equal values. For example, it is not possible to add  $\frac{1}{3}$  to  $\frac{2}{5}$  until the denominators of the fractions have been changed to equal values. Since 3 and 5 will both go evenly into 15, we can change  $\frac{1}{3}$  to  $\frac{5}{15}$ , and  $\frac{2}{5}$  to  $\frac{6}{15}$ . In this case, 15 is called the **lowest common denominator (LCD)** of the fractions being considered. It is now a simple matter to add the fractions.

$$\frac{5}{15} + \frac{6}{15} = \frac{11}{15}$$

We can see that this addition makes sense because  $5 + 6 = 11$ . Since both the 5 and the 6 denote a specific number of fifteenths, we add them to obtain the total number of fifteenths.

The foregoing principle may be understood more easily if we apply it to a practical problem. Suppose we wish to add 3 gal and 5 qt and 1 pint (pt) of gasoline. The most logical method is to convert all quantities to pints. In 3 gal of gasoline there are  $3 \times 8$  or 24 pt; in 5 qt there are  $5 \times 2$  or 10 pt. Then we add 24 pt + 10 pt + 1 pt. The answer is 35 pt. If we wish to convert this quantity to gallons, we must divide the 35 by 8. We find that we have 4 gal and 3 pt, or  $4\frac{3}{8}$  gal.

To prepare fractions for adding or subtracting we proceed as follows:

1. Find the LCD.
2. Divide the LCD by each denominator.
3. Multiply the numerator and denominator of each fraction by the quotient obtained when the LCD was divided by the denominator.

To practice these steps, perform the following addition:

$$\frac{3}{4} + \frac{7}{8} + \frac{5}{6}$$

The LCD is 24. Divide the LCD by the first denominator, and then multiply the fraction by this quotient:

$$24 \div 4 = 6$$

$$\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$

Do the same for the second fraction:

$$24 \div 8 = 3$$

$$\frac{7 \times 3}{8 \times 3} = \frac{21}{24}$$

And for the third fraction:

$$24 \div 6 = 4$$

$$\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$

Then add all the fractions:

$$\frac{18}{24} + \frac{21}{24} + \frac{20}{24} = \frac{59}{24} = 2\frac{11}{24}$$

### Adding and Subtracting Mixed Numbers

When adding and subtracting mixed numbers, we must consider both the whole numbers and the fractions. To add  $5\frac{3}{8} + 7\frac{2}{3}$ , we should first add 5 and 7 to obtain 12, and then we must add the fractions. We find that  $\frac{3}{8} = \frac{9}{24}$ ,  $\frac{2}{3} = \frac{16}{24}$ , and thus  $\frac{9}{24} + \frac{16}{24} = \frac{25}{24}$ , or  $1\frac{1}{24}$ . Then  $12 + 1\frac{1}{24} = 13\frac{1}{24}$ , the total sum of the mixed numbers.

Subtraction of mixed numbers is accomplished by subtracting the whole numbers and then the fractions. For example, subtract  $8\frac{2}{3}$  from  $12\frac{3}{4}$ .

$$\begin{array}{r} 12\frac{3}{4} = 12\frac{9}{12} \\ -8\frac{2}{3} = -8\frac{8}{12} \\ \hline 4\frac{1}{12} \end{array}$$

If the fraction of the subtrahend is greater than the fraction of the minuend, it is necessary to borrow 1 from the whole number in the minuend to increase the fraction of the minuend. If we wish to subtract  $5\frac{7}{8}$  from  $9\frac{1}{3}$ , we must increase the  $\frac{1}{3}$  to a value greater than  $\frac{7}{8}$ . The LCD of the fractions is 24, and so  $5\frac{7}{8}$  becomes  $5\frac{21}{24}$  and  $9\frac{1}{3}$  becomes  $9\frac{8}{24}$ . We must then borrow 1 from 9 and add the 1 to  $\frac{8}{24}$ . The minuend then becomes  $8\frac{32}{24}$ . The final form of the problem is then

$$\begin{array}{r} 8\frac{32}{24} \\ -5\frac{21}{24} \\ \hline 3\frac{11}{24} \end{array}$$

### Multiplication of Fractions

Multiplication of fractions is accomplished by placing the product of the numerators over the product of the denominators. This result is then reduced to lowest terms. For example,

$$\frac{2}{5} \cdot \frac{1}{2} \cdot \frac{3}{4} = \frac{6}{40} = \frac{3}{20}$$

Where possible in the multiplication of fractions, cancellation is employed to simplify the fractions before final multiplication takes place.

$$\frac{\cancel{2}}{5} \cdot \frac{\cancel{1}}{\cancel{2}} \cdot \frac{3}{10} \cdot \frac{\cancel{4}}{\cancel{2}} = \frac{3}{10}$$